Supported by tetfund

CONSTRUCTION OF BETA-TOPP LEONE DISTRIBUTION

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Received: December 16, 2019 **Accepted:** February 19, 2020

Abstract: In this research work, a continuous probability distribution as an improvement of Topp-Leone distribution was developed. Thus, density function and the cumulative function of the constructed distribution were obtained. We also show the validity of the new distribution. However, some properties of constructed distribution such as, survival and hazard functions, moment, and probability weighted moment and order statistics were derived. Parameters were obtained using method of maximum likelihood estimation.

Keywords: Beta-G family, Topp-Leone distribution, Beta-Topp Leone, density function, moments

Introduction

Probability distribution theory, preference for a particular probability distribution in modeling real life phenomena could be based on either the distribution is tractable or the distribution is flexible (Oguntunde *et al.*, 2016).

The tractability of a probability distribution may be useful in theory because such distribution would be easy to work with; especially when it comes to simulation of random samples, but to practitioners and some other stakeholders, the flexibility of probability distributions could be of interest. In fact, it is preferable to make use of probability distributions that best fit the available data set than to transform the existing data set as this may affect the originality of the data set. Because of this, several efforts have been made in recent years to ensure that the existing standard theoretical distributions are modified and extended (Merovci, 2013); this could increase their flexibility and enhance the capability to model real life data sets.

To extend an existing standard distribution, there are various approaches that could be adopted. For instance, the flexibility of a distribution can be increased by means of generalization which involves using the available generalized family of distributions.

When a distribution is generalized, extra shape parameter(s) from the family of distributions used would have been added. The role of these additional shape parameter(s) is to vary the tail weight of the resulting compound distribution, thereby inducing it with skewness. The flexibility can also be increased by modifying the existing distribution. For instance, two or more standard distributions can be combined as in the case of convolution, quotient, or product of independent random variables. Also, some distributions are found to be functions of one or more other distributions; the composition of the student t-distribution is a common example (Sun, 2011).

Akinsete *et al.* (2008) defined beta-Pareto distribution. The authors discussed various properties of the distribution. Their distribution was found to be unimodal and has either a unimodal or a decreasing hazard rate. The expressions for the mean, mean deviation, variance, skewness, kurtosis and entropies were obtained. The relationship between these moments and the parameters were provided. The method of maximum likelihood was also proposed to estimate the parameters of the distribution. The distribution was applied to two flood data sets.

Hanook *et al.* (2013) derived densities and various statistical properties of the Beta Inverse Weibull distribution. The relationships that exist between its parameters, mean, skewness, variance and kurtosis were also investigated. The shape of the distribution was unimodal and the model parameters were successfully estimated.

Though, an application to real life data sets was not provided but the authors claimed that the Beta Inverse Weibull distribution would receive wider attraction in reliability and mechanical engineering.

Jafari *et al.* (2014) gave the densities and properties of the Beta Gompertz distribution. Distributions like the Exponential distribution, Generalised Exponential distribution, Generalised Gompertz distribution, Gompertz distribution and the Beta Exponential distribution were discovered to be special cases of the Beta Gompertz distribution. A simulation study was conducted to investigate the generalization of the proposed distribution. An application to a data set on lifetime of 50 devices revealed that the Beta Gompertz distribution has a better fit than all its sub-models.

Nekoukhou (2016) studied a discrete analog of the beta-Rayleigh distribution. The distribution contains the generalized discrete Rayleigh and discrete Rayleigh distributions as special sub-models. The author discussed some distributional and moment properties of the new discrete distribution as well as its order statistics. So, the hazard rate function of his model can be increasing, bathtub-shaped and upside-down bathtub. Estimation of the parameters was illustrated.

Usman *et al.* (2019) constructed a two parameters continuous distribution using Topp-Leone distribution as a baseline. They derived density function and the cumulative function of the constructed distribution (Burr X-Topp Leone distribution). Also, they show the validity of the new distribution.

Methodology

Following the family proposed by Eugene *et al.* (2002). Supposed $F(x)$ and $f(x)$ are the cumulative density function (cdf) and probability density function (pdf) of any baseline distribution, then, the cdf and the pdf of the beta generalized family is given by equation 1 and 3.

$$G(x) = I_{G(x)}(a,b) \qquad 1$$

Where: $I_{G(x)}(a,b)$ is regarded as the incomplete beta function ratio

Mathematically,

$$I_{G(x)}(a,b) = \frac{1}{B(a,b)}\int_0^{F(x)} x^{a-1}(1-x)^{b-1}dx \qquad 2$$

(Sangsanit & Bodhisuwan, 2016)

And

$$g(x) = \frac{1}{B(a,b)}[F(x)]^{a-1}[1-F(x)]^{b-1}f(x) \qquad 3$$

Where: $f(x)$ and $F(x)$ denote the pdf and cdf of baseline model

In this paper, Topp Leone Distribution is considered as a baseline distribution. If a random variable T is distributed as the Topp-Leone and bounded on [0,1] (Sangsanit & Bodhisuwan, 2016). Let X be a continuous random variable with pdf $f(x)$. TL distribution has pdf written by;

$$f(x) = \beta(2-2x)(2x-x^2)^{\beta-1} \quad 4$$

Where $\beta > 0$ is a shape parameter. The associated cdf is;

$$F(x) = (2x-x^2)^{\beta} \quad 5$$

Proposed beta-topp leone distribution (BTLD)

Using the idea of Eugene *et al.*, (2002), we propose a new continuous probability distribution called the Beta-Topp Leone Distribution (BTLD).Thus, to obtain the pdf and the cdf of the new model, we substitute our baseline distribution in equation 3 and 5. So that:

$$g(x)_{BTLD} = \frac{1}{B(a,b)}[\beta(2-2x)(2x-x^2)^{\beta a-1}][1-(2x-x^2)^{\beta}]^{b-1} \quad 6$$

is the pdf with 3 parameters.

And

$$G(x)_{BTLD} = \frac{1}{B(a,b)}\int_0^{(2x-x^2)^{\beta}} x^{a-1}(1-x)^{b-1}\,dx = \frac{\beta(2x-x^2)(a,b)}{B(a,b)} \text{ is the cdf} \quad 7$$

For $0 \le x \le 1, \beta > 0$, where $a > 0$ *and* $b > 0$ are the shape parameters.

Model validity check of the proposed BTLD

A probability distribution is a function that describes the likelihood of obtaining the possible values that a random variable can assume. In other words, the values of the variable were based on the underlying probability distribution (Jim, 2019). As such, a function can serve as a probability function of a continuous random variable X if its values, $f(x)$ satisfies the conditions:

$f(x) \ge 0$, for $-\infty < x < \infty$

$$\int_{-\infty}^{\infty} g(x)dx = 1 \quad 8$$

It follows from the above that if X is a continuous random variable, then the probability that X takes on any one particular value is zero, whereas the interval probability that X lies between two different values, say, a and b, is given by

$$P(a < X < b) = \int_a^b g(x)dx = 1$$

Thus, with the limit of the BTLD (0,1), we have;

$$P(0 < X < 1) = \int_0^1 g(x)_{BTLD}dx = 1 \quad 9$$

From 6 and 9, we have:

$$\int_0^1 \frac{1}{B(a,b)}[\beta(2-2x)(2x-x^2)^{\beta a-1}][1-(2x-x^2)^{\beta}]^{b-1}\,dx = 1 \quad 10$$

$$\frac{\beta}{B(a,b)}\int_0^1 (2-2x)(2x-x^2)^{\beta a-1}\,[1-(2x-x^2)^{\beta}]^{b-1}dx = 1 \quad 11$$

Let $y = 2x - x^2, \frac{dy}{dx} = 2-2x => dx = \frac{dy}{(2-2x)}$

Using the above terms in 11, we obtain:

$$\frac{\beta}{B(a,b)}\int_0^1 y^{\beta a-1}(1-y^{\beta})^{b-1}\,dy = 1 \quad 12$$

Let $z = y^{\beta}, dz = (\beta y^{\beta-1})dy, dy = \frac{dz}{\beta y^{\beta-1}}$

From the above terms, the equation follows:

$$\frac{1}{B(a,b)}\int_0^1 z^{\frac{\beta a-1}{\beta}}(1-z)^{b-1}\frac{dz}{y^{\beta-1}} = 1 \quad 13$$

$$\frac{1}{B(a,b)}\int_0^1 z^{a-\frac{1}{\beta}}z^{-1+\frac{1}{\beta}}(1-z)^{b-1}dz = 1 \quad 14$$

$$\frac{1}{B(a,b)}\int_0^1 z^{a-1}(1-z)^{b-1}\,dz = 1 \quad 15$$

Since $B(a,b)$ is also know as the normalizing constant because it makes the integral equal to 1

Thus,

$$\int_0^1 Z^{a-1}(1-Z)^{b-1}\,dz = B(a,b) \quad 16$$

Then

$\frac{1}{B(a,b)} \times B(a,b) = 1$, hence the proof.

However, since $\int_0^1 g(x)_{BTLD}dx = 1$, it means the proposed BTLD is valid.

Reliability analysis of the BTLD

The task of survival is the possibility that the system or person does not fail after a certain time. The task of survival is given:

$$S(X) = P(X > x) = 1 - G(x) \quad 17$$

Applying the BTLD in (3.7), the survival function for the BTLD is obtained as:

$$S(x) = \frac{B(a,b) - \beta(2x-x^2)(a,b)}{B(a,b)} \quad 18$$

Hazard function is the probability that a component will fail or die for an interval of time. The hazard function is defined as;

$$h(x) = \frac{f(x)}{S(x)} = \frac{g(x)}{1-G(x)} \quad 19$$

$$h(x) = \frac{\beta(2-2x)(2x-x^2)^{\beta a-1}[1-(2x-x^2)^{\beta}]^{b-1}}{B(a,b) - \beta G(x)(a,b)} \quad 20$$

Linear representation

In this section, we derive expansions for the cdf and pdf of the BTLD that are useful to study its statistical properties using an idea by Shittu and Adepoju (2013).

Applying 4 and 5 into 3, we have:

$$g(x)_{BTLD} = \frac{\beta(2-2x)(2x-x^2)^{\beta-1}}{B(a,b)}[(2x-x^2)^{\beta}]^{a-1}[1-(2x-x^2)^{\beta}]^{b-1} \quad 21$$

Consider the series expansion

$$[1-(2x-x^2)^{\beta}]^{b-1} = \sum_{i=0}^{\infty}\binom{b-1}{i}(-1)^i(2x-x^2)^{\beta i} \quad 22$$

Where, the binomial coefficient is defined for any real number.

Thus, 21 becomes

$$= \frac{\beta(2-2x)(2x-x^2)^{\beta-1}}{B(a,b)}[(2x-x^2)^{\beta}]^{a-1}\sum_{i=0}^{\infty}(-1)^i\,(2x-x^2)^{\beta i} \quad 23$$

We can expand $[(2x-x^2)^{\beta}]^{a-1}$ as follows:

$$\left[1-[1-(2x-x^2)^{\beta}]\right]^{a-1} = \sum_{m=0}^{\infty}\binom{a-1}{m}(-1)^m[1-(2x-x^2)^{\beta}]^m \quad 24$$

Also,

$$[1-(2x-x^2)^{\beta}]^m = \sum_{n=0}^{\infty}\binom{m}{n}(-1)^n(2x-x^2)^{\beta n} \quad 25$$

Applying 24 and 25 in 21, we have:

$$= \frac{\beta(2-2x)(2x-x^2)^{\beta-1}}{B(a,b)}\sum_{i,m,n=0}^{\infty}\binom{b-1}{i}\binom{a-1}{m}\binom{m}{n}(-1)^{i+m+n}(2x-x^2)^{\beta i}(2x-x^2)^{\beta n} \quad 26$$

$$= \frac{\beta(2-2x)(2x-x^2)^{\beta-1}(k+1)}{B(a,b)(k+1)}\sum_{i,m,n=0}^{\infty}\binom{b-1}{i}\binom{a-1}{m}\binom{m}{n}(-1)^{i+m+n}(2x-x^2)^{\beta(i+n)} \quad 27$$

Let

$$\frac{\sum_{i,m,n=0}^{\infty}\binom{b-1}{i}\binom{a-1}{m}\binom{m}{n}(-1)^{i+m+n}}{B(a,b)(k+1)} = \emptyset_{i,m,n}(a,b) \quad 28$$

And

$$\beta(2-2x)(2x-x^2)^{\beta-1}(k+1)(2x-x^2)^{\beta(i+n)} = \pi_k(x) \quad 29$$

Thus,

$$g(x) = \emptyset_{i,m,n}(a,b)\pi_k(x) \quad 30$$

Moment

Let X denote a continuous random variable, the S[th] moment of X is given by:

$$E(x^s) = \int_{-\infty}^{\infty} x^s\,g(x)dx \quad 31$$

$$= \int_{-\infty}^{\infty} \emptyset_{i,m,n}(a,b)\pi_k(x)dx \quad 32$$

$$= (k+1)\emptyset_{i,m,n}(a,b)\int_0^1 x^s\beta(2x-2x)(2x-x^2)^{\beta-1}(2x-x^2)^{\beta(i+n)}dx \quad 33$$

Let $U = (k+1)\emptyset_{i,m,n}(a,b)$

Therefore, we have

$$U\int_0^1 x^s\beta(2-2x)(2x-x^2)^{\beta[[i+n]+1]-1}dx \quad 34$$

Let consider

$$\left[1-[1-(2x-x^2)]\right]^{\beta[[i+n]+1]-1}=\sum_{q,v=0}^{\infty}\binom{\beta[[i+n]+1]-1}{q}\binom{q}{v}(-1)^{q+v}(2x-x^2)^v \quad 35$$

Substitute 35 in to 34, it gives:

$$=U\sum_{q,v=0}^{\infty}\binom{\beta[[i+n]+1]-1}{q}\binom{q}{v}(-1)^{q+v}\int_0^1 x^s\beta(2-2x)(2x-x^2)^v dx \quad 36$$

$$=2\beta U\sum_{q,v=0}^{\infty}\binom{\beta[[i+n]+1]-1}{q}\binom{q}{v}(-1)^{q+v}\int_0^1 x^{s+v}(1-x)(2-x)^v dx \quad 37$$

Let $Q=2\beta U\sum_{q,v=0}^{\infty}\binom{\beta[[i+n]+1]-1}{q}\binom{q}{v}(-1)^{q+v}$ 38

$$and\ y=1-x, x=1-y, dx=-dy$$

Substituting above terms in to equation 43 gives

$$=Q\int_1^0(1-y)^{s+v}y(1+y)^v-dy \quad 39$$

Where,

$$(1+y)^v=\sum_w^{\infty}(-1)^w\binom{v}{w}y^w \quad 40$$

Using 40 gives:

$$=Q\sum_w^{\infty}(-1)^w\binom{v}{w}\int_0^1(1-y)^{s+v}y^{w+1}dy \quad 41$$

Thus,

$$=Q\sum_w^{\infty}(-1)^w\binom{v}{w}\int_0^1(1-y)^{(s+v+1)-1}y^{(w+2)-1}dy \quad 42$$

$$=Q\frac{\Gamma[(s+v+1)-1]\Gamma(w+2)-1}{\Gamma[[(s+v+1)-1]+[(w+2)-1]]} \quad 43$$

Mean

The mean of the BTLD can be obtained from Sth moment of the distribution when s=1 as follows:

$$\mu_1=E(X)=Q\frac{\Gamma[(v+2)-1]\Gamma(w+2)-1}{\Gamma[[(v+2)-1]+[(w+2)-1]]} \quad 44$$

Also the second moment of BTLD is obtained from the sth moment of the distribution when s=2:

$$\mu_2=E(X^2)=Q\frac{\Gamma[(v+2)-1]\Gamma(w+2)-1}{\Gamma[[(v+2)-1]+[(w+2)-1]]} \quad 45$$

Variance

$$Var(X)=E(X^2)-[E(X)]^2 \quad 46$$

$$=Q\frac{\Gamma[(v+2)-1]\Gamma(w+2)-1}{\Gamma[[(v+2)-1]+[(w+2)-1]]}-\left[Q\frac{\Gamma[(v+2)-1]\Gamma(w+2)-1}{\Gamma[[(v+2)-1]+[(w+2)-1]]}\right]^2 \quad 47$$

Probability weighted moment (PWM)

The PWM of X following the Beta-G distribution is formally defined by:

$$g(x)_{BTLD}=\int_{-\infty}^{\infty}x^s[G(x)]^r g(x)dx \quad 48$$

Applying 29 in to 48

$$=\int_{-\infty}^{\infty}x^s[G(x)]^{\beta(i+n)}g(x)dx \quad 49$$

$$=\int_0^1 x^s\beta(2x-2x)(2x-x^2)^{\beta-1}\left[(2x-x^2)^{\beta}\right]^{\beta(i+n)}dx \quad 50$$

Consider

$$\left[1-\left[1-(2x-x^2)^{\beta}\right]\right]^{\beta(i+n)}=\sum_{i=0}^{\infty}(-1)^i\binom{\beta(i+n)}{i}\left[1-(2x-x^2)^{\beta}\right]^i \quad 51$$

Also,

$$\left[1-(2x-x^2)^{\beta}\right]^i=\sum_{j=0}^{\infty}(-1)^j\binom{i}{j}(2x-x^2)^{\beta j}$$

52

Using 51 and 52 into 50 leads to:

$$=\int_0^1 x^s\beta(2-2x)(2x-x^2)^{\beta-1}\sum_{i,j=0}^{\infty}(-1)^{j+j}\binom{\beta(i+n)}{i}\binom{i}{j}(2x-x^2)^{\beta j}dx \quad 53$$

$$=\sum_{i,j=0}^{\infty}(-1)^{j+j}\binom{\beta(i+n)}{i}\binom{i}{j}\int_0^1 x^s\beta(2-2x)(2x-x^2)^{\beta(j+1)-1} \quad 54$$

Where

$$(2x-x^2)^{\beta(j+1)-1}=\sum_{k=0}^{\infty}(-1)^k\binom{\beta(j+1)-1}{k}(2x)^{[\beta(j+1)-1]-k}x^{2k} \quad 55$$

Then, put 55 in 54

$$2^{[\beta(j+1)-1]-k}\beta\sum_{i,j,k=0}^{\infty}(-1)^{j+j+k}\binom{\beta(i+n)}{i}\binom{i}{j}\binom{\beta(j+1)-1}{k}\int_0^1 x^{s+[\beta(j+1)-1]+k}(2-2x)dx \quad 56$$

Order statistics

Suppose $X_1,X_2,\dots,X_n$ is a random sample from any BTLD. Let $X_r{:}n$ denote the order statistics. The pdf of $X_r{:}n$ can be expressed as:

$$f_r{:}n(X,a,b,\beta)=\frac{n!}{(i-1)!(n-i)!}f(X)^{BTLD}[F(X)^{BTLD}]^{i-1}[1-F(X)^{BTLD}]^{n-1} \quad 57$$

Substitute 6 and 7 in 57

$$=\frac{n!\beta(2-2x)(2x-x^2)^{\beta a-1}\left[1-(2x-x^2)^{\beta}\right]^{b-1}}{(i-1)!(n-i)!B(a,b)}\left[\frac{\beta(2x-x^2)(a,b)}{B(a,b)}\right]^{i-1}\left[1-\left[\frac{\beta(2x-x^2)(a,b)}{B(a,b)}\right]\right]^{n-i} \quad 58$$

Consider

$$\left[1-\left[\frac{\beta(2x-x^2)(a,b)}{B(a,b)}\right]\right]^{n-i}=\sum_i^{\infty}(-1)^i\binom{n-i}{i}\left[\frac{\beta(2x-x^2)(a,b)}{B(a,b)}\right]^i \quad 59$$

and

$$\left[1-(2x-x^2)^{\beta}\right]^{b-1}=\sum_{j=0}^{\infty}(-1)^j\binom{b-1}{j}(2x-x^2)^{\beta j} \quad 60$$

Then 58 becomes

$$=\frac{n!\sum_{i,j=0}^{\infty}(-1)^{i+j}\binom{n-1}{i}\binom{b-1}{j}}{(i-1)!(n-i)!B(a,b)}\beta(2-2x)(2x-x^2)^{\beta(a+j)-1}\left[\frac{\beta(2x-x^2)(a,b)}{B(a,b)}\right]^{2i-1} \quad 61$$

If

$$(2x-x^2)^{\beta(a+j)-1}=\sum_{k=0}^{\infty}(-1)^k\binom{\beta(a+j)-1}{k}(2x)^{[\beta(a+j)-1]-k}x^{2k}$$

Then

$$=\frac{2^{[\beta(a+j)-1]-k}\beta n!\sum_{i,j,k=0}^{\infty}(-1)^{i+j+k}\binom{n-1}{i}\binom{b-1}{j}\binom{\beta(a+j)-1}{k}}{(i-1)!(n-i)!B(a,b)}(2-2x)x^{[\beta(a+j)-1]+k}\left[\frac{\beta(2x-x^2)(a,b)}{B(a,b)}\right]^{2i-1} \quad 62$$

Let $\frac{2^{[\beta(a+j)-1]-k}\beta n!\sum_{i,j,k=0}^{\infty}(-1)^{i+j+k}\binom{n-1}{i}\binom{b-1}{j}\binom{\beta(a+j)-1}{k}}{(i-1)!(n-i)!B(a,b)}=Q$

Then

$$Q(2-2x)x^{[\beta(a+j)-1]+k}\left[\frac{\beta(2x-x^2)(a,b)}{B(a,b)}\right]^{2i-1} \quad 63$$

Parameter estimation

The model parameters of the BTLD can be estimated by maximum likelihood. Let $X=(x_1,x_2,\dots,x_n)'$ be a random sample of size n from BTL with parameter vector $\theta=(a,b,\beta)'$.

Then, the likelihood function can be expressed as:

$$L(\theta)=\prod_{i=1}^{n}f(x)_{BTLD} \quad 64$$

Using equation 9 in 75, that is the pdf of BTLD, we have:

$$=[\beta(a,b)]^{-n}\left[\beta^n\sum_{i=1}^{n}(2-2x)(2x-x^2)^{\beta a-1}\right]\sum_{i=1}^{n}\left[1-(2x-x^2)^{\beta}\right]^{b-1} \quad 65$$

The log-likelihood function is

$$l(\theta) = nlog\beta + \sum_{i=1}^{n} log(2-2x) + (\beta a - 1)\sum_{i=1}^{n} \log(2x - x^2) + (b-1)\sum_{i=1}^{n} log\left[1 - (2x - x^2)^{\beta}\right] - nlog[\beta(a,b)] \quad 66$$

By taking the partial differentiation of $l(\theta)$ with respect to a, b, β respectively, the components of the unit score vector $\mu_\theta = \left(\mu_a, \mu_b and\ \mu_\beta\right)'$ can be obtained as follows:

$$\mu_a = \frac{dl}{da} = \sum_{i=0}^{n} \log(2x - x^2)\beta \frac{-n}{\beta(a,b)} d\beta(a,b) \quad 67$$

$$= \beta \sum_{i=1}^{n} log\,(2x - x^2) - \frac{nd\beta(a,b)}{\beta(a,b)}$$

$$= \beta \sum_{i=1}^{n} log\,(2x - x^2) - n\varphi(a) \quad 68$$

$$\mu_b = \frac{dl}{db} = \sum_{i=1}^{n} log\left[1 - (2x - x^2)^{\beta}\right] - \frac{n}{\beta(a,b)} d\beta(a,b) \quad 69$$

$$= \sum_{i=1}^{n} log\left[1 - (2x - x^2)^{\beta}\right] - \frac{nd\beta(a,b)}{\beta(a,b)}$$

$$= \sum_{i=1}^{n} log\left[1 - (2x - x^2)^{\beta}\right] - n\varphi(b) \quad 70$$

$$\mu_\beta = \frac{dl}{d\beta} = \frac{n}{\beta} + \sum_{i=1}^{n} log(2x - x^2)\,a + (1 - b)\frac{\sum_{i=1}^{n}(2x-x^2)^{\beta}\log(2x-x^2)}{\left[1-(2x-x^2)^{\beta}\right]}(-1) \quad 71$$

$$= \frac{n}{\beta} + a\sum_{i=1}^{n} log(2x - x^2) + (1 - b)\frac{\sum_{i=1}^{n}(2x-x^2)^{\beta} log(2x-x^2)}{\left[1-(2x-x^2)^{\beta}\right]} \quad 72$$

However, log-likelihood function of this distribution cannot be solved analytically because of its complex form but it can be maximized by employing global optimization methods available with softwares like R software, SAS, Mathematical and so on.

Conclusion

In this study, we constructed a continuous probability distribution with three shape parameters. The density function and the cumulative function of the constructed distribution were derived. Also, the validity of the new distribution was illustrated. Some statistical properties of the proposed distribution which include reliability analysis, moments, probability weighted moment and order statistics have been presented. The method of maximum likelihood was used to estimate the parameters.

Conflict of Interest

Authors declare there is no conflict of interest related to this study.

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